## Mixture Problems

Here are a few tips to remember when solving mixture problems:

1. Construct a table showing the given information. It may also be helpful to sketch a diagram similar to the diagram below in order to illustrate the given quantities.
2. Use the variable $\mathbf{x}$ to represent the unknown volume or amount of a mixture.
3. The volume of the first mixture plus the volume of the second mixture will be equal to the total volume of the final mixture.
4. The equation will be obtained by summing the volume of pure substance contained in each mixture, and setting this sum equal to the volume of pure substance contained in the final mixture. To find the volume of pure substance contained in a mixture, multiply the given percentage (\%) by the corresponding volume of the mixture.

Example: A mixture containing 6\% boric acid is to be mixed with 2 quarts of a mixture that is $15 \%$ acid, in order to obtain a solution that is $12 \%$ acid. How much of the $6 \%$ solution must be used?

## Draw a diagram



Let $\mathbf{x}=$ quarts of $6 \%$ solution.

|  | Percentage (\%) | Volume (quarts) | Pure Substance $=\%$ * quarts |
| :--- | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Mixture | 6 | x | 6 x |
| $\mathbf{2}^{\mathbf{n d}}$ Mixture | 15 | 2 | $15(2)$ |
| Final Mixture | 12 | $\mathrm{x}+2$ | $12(\mathrm{x}+2)$ |

## Explanation:

The amount of pure substance (last column) in each mixture is determined by multiplying each percentage by its corresponding volume. The equation is then formed by adding the first two entries of the "Pure Substance" column and setting this sum equal to the last entry of that column.

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Equation: \(\quad 6 x+15(2)=12(x+2)\)
    \(6 x+30=12 x+24\)
    \(-6 x=-6\)
    \(\mathrm{x}=1\)
```

1 quart of the 6\% solution must be used.

## Sample Problems:

1. If alloy containing $20 \%$ silver is mixed with pure silver to get 800 pounds of $40 \%$ alloy, how much of the $20 \%$ alloy and pure silver must be used?


Let $\mathbf{x}=$ pounds of $20 \%$ alloy.

|  | Percentage (\%) | Volume (pounds) | Pure Substance $=\%$ pounds |
| :--- | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Mixture | 20 | X | 20 x |
| $\mathbf{2}^{\text {nd }}$ Mixture (pure) | $\mathbf{1 0 0}$ | $800-\mathrm{x}$ | $100(800-\mathrm{x})$ |
| Final Mixture | 40 | 800 | $40(800)$ |

Equation:

$$
\begin{aligned}
& 20 x+100(800-x)=40(800) \\
& 20 x+80000-100 x=32000 \\
& -80 x+80000=32000 \\
& -80 x=-48000 \\
& x=600
\end{aligned}
$$

Since x represents the volume of the $20 \%$ alloy, we need $\mathbf{6 0 0}$ pounds of $20 \%$ alloy. The volume of pure silver needed is represented by $800-\mathrm{x}$. We will therefore need 200 pounds of pure silver.

## Comment: 800-x=800-600

2. Dr. Lytle orders 20 grams of a $52 \%$ solution of a certain medicine. The pharmacist has only bottles of $40 \%$ and bottles of $70 \%$ solution. How much of each must be used to obtain the 20 grams of the $52 \%$ solution?

Let $\mathbf{x}=$ grams of the $40 \%$ solution.

|  | Percentage (\%) | Volume (grams) | Pure Substance $=\%$ * grams |
| :--- | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Mixture | 40 | X | 40 x |
| $\mathbf{2}^{\text {nd }}$ Mixture | 70 | $20-\mathrm{x}$ | $70(20-\mathrm{x})$ |
| Final Mixture | 52 | 20 | $52(20)$ |

Equation:

$$
\begin{aligned}
& 40 x+70(20-x)=52(20) \\
& 40 x+1400-70 x=1040 \\
& -30 x+1400=1040 \\
& -30 x=-360 \\
& x=12
\end{aligned}
$$

12 grams of the $40 \%$ solution must be used, since $x$ represents the volume of $40 \%$ solution. The volume of the $70 \%$ solution that must be used is $\mathbf{8}$ grams. $\square$
3. Bryan discovers at the end of the summer that his radiator antifreeze solution has dropped below the safe level. If the radiator contains 4 gallons of a $25 \%$ solution, how many gallons of pure antifreeze must he add to bring it up to a desired $50 \%$ solution?

Let $\mathbf{x}=$ gallons of pure antifreeze.

|  | Percentage (\%) | Volume (gallons) | Pure Substance $=\%$ * gallons |
| :--- | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Mixture | 25 | 4 | $25(4)$ |
| $\mathbf{2}^{\text {nd }}$ Mixture (pure) | $\mathbf{1 0 0}$ | x | 100 x |
| Final Mixture | 50 | $\mathrm{x}+4$ | $50(\mathrm{x}+4)$ |

Equation: $\quad 25(4)+100 x=50(x+4)$
$100+100 \mathrm{x}=50 \mathrm{x}+200$
$50 \mathrm{x}=100$
$\mathrm{x}=2$
He will need to add 2 gallons of pure antifreeze to obtain the desired $50 \%$ solution.
4. Ms. Hardy has 25 ounces of a $20 \%$ boric acid solution that she wishes to dilute to a $10 \%$ solution. How much water does she have to add in order to obtain the $10 \%$ solution?

Let $\mathbf{x}=$ ounces of water. Since water contains no boric acid, we will use $\mathbf{0}$ to represent the percentage for water.

|  | Percentage (\%) | Volume (ounces) | Pure Substance $=\%$ * ounces |
| :--- | :---: | :---: | :---: |
| $\mathbf{1}^{\text {st }}$ Mixture | 20 | 25 | $20(25)$ |
| $\mathbf{2}^{\text {nd }}$ Mixture | $\mathbf{0}$ | x | 0 x |
| Final Mixture | 10 | $\mathrm{x}+25$ | $10(\mathrm{x}+25)$ |

Equation:

$$
\begin{aligned}
& 20(25)+0 x=10(x+25) \\
& 500=10 x+250 \\
& -10 x=-250 \\
& x=25
\end{aligned}
$$

She should add 25 ounces of water to obtain the $10 \%$ solution.
5. A candy shop owner wishes to sell a bag containing two different kinds of candy. If Candy A costs $\$ 0.40$ per pound and Candy B costs $\$ 0.70$ per pound, how many pounds of Candy A must the shop owner add to 6 pounds of Candy B, if he wishes to sell the mixture for $\$ 0.55$ per pound?

Let $\mathbf{x}=$ pounds of Candy A.

|  | Cost (\$) per pound | Volume (pounds) | Total Cost $=\$ *$ pounds |
| :--- | :---: | :---: | :---: |
| Candy A | .40 | x | .40 x |
| Candy B | .70 | 6 | $.70(6)$ |
| Final Mixture | .55 | $\mathrm{x}+6$ | $.55(\mathrm{x}+6)$ |

Equation:

$$
\begin{aligned}
& .40 \mathrm{x}+.70(6)=.55(\mathrm{x}+6) \\
& .40 \mathrm{x}+4.2=.55 \mathrm{x}+3.3 \\
& -.15 \mathrm{x}=-.9 \\
& \mathrm{x}=6
\end{aligned}
$$

The shop owner should add 6 pounds of Candy A to Candy B.

