ABSOLUTE VALUE INEQUALITIES

Step 1: Isolate the absolute value expression
Step 2: STOP! If the constant value is negative, you have a special case (see page 2)
Step 3: Replace the inequality sign with an equal sign (=)
Step 4: Set up 2 equations (one positive, one negative) to create **critical points**
Step 5: Graph the critical points on a number line and test a number in each zone to determine the solution set

Example: Solve $|1-3x|-4 \geq 3$

Step 1: $|1-3x|-4+4 \geq 3+4$ add 4 to each side to isolate the absolute value

$$|1-3x| \geq 7$$

Step 2: STOP and check! 7 is positive, so we do not have a special case

Step 3: Replace the inequality sign with the equal sign

$$|1-3x| = 7$$

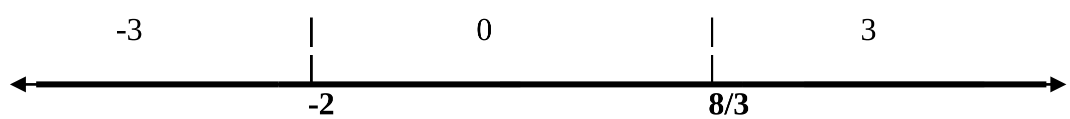
Step 4: Remove the absolute value bars and set up 2 equations (one positive, one negative)

$$\begin{array}{ccc} 1-3x=7 & \textbf{OR} & 1-3x=-7 \\ 1-3x-1=7-1 & & 1-3x-1=-7-1 \\ -3x=6 & & -3x=-8 \\ \frac{-3x}{-3}=\frac{6}{-3} & & \frac{-3x}{-3}=\frac{-8}{-3} \\ \boldsymbol{x=-2} & & \boldsymbol{x=\frac{8}{3}} \end{array}$$

So, there are 2 critical points: $x=-2$ and $x=\frac{8}{3}$

Step 5:

Zone A	**Zone B**	**Zone C**



$\lvert 1-3(-3)\rvert - 4 \geq 3$	$\lvert 1-3(0)\rvert - 4 \geq 3$	$\lvert 1-3(3)\rvert - 4 \geq 3$
$\lvert 10\rvert - 4 \geq 3$	$\lvert 1\rvert - 4 \geq 3$	$\lvert -8\rvert - 4 \geq 3$
$10 - 4 \geq 3$	$1 - 4 \geq 3$	$8-4 \geq 3$
$6 \geq 3$	$-3 \geq 3$	$4 \geq 3$
True	False	True

Use the Zones that test "True" to build the solution set. Use square brackets on the end points to show that the end points are included in the solution set.

Graph of Solution Set (interval notation style**):**

Interval Notation: $(-\infty, -2] \cup [\frac{8}{3}, \infty)$ **Set Builder Notation**: $\{x \mid x \leq -2 \text{ or } x \geq \frac{8}{3}\}$

Special Cases

<u>Example #1</u>: Solve $|7x + 8| + 5 < 2$

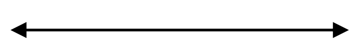
Step 1: $|7x + 8| + 5 - 5 < 2 - 5$

$$|7x + 8| < -3$$

<u>STOP!</u>

Step 2: An absolute value **<u>always</u>** gives a **<u>positive</u>** answer, so it can never be less than a negative 3. NO values of *x* will make this inequality true, so the solution set is empty.

Graph of Solution Set: (<u>blank line</u>)



Interval Notation: $\emptyset$

Set Builder Notation: { }

<u>Example #2</u>: Solve $|x| + 10 > 4$

Step 1: $|x| + 10 - 10 > 4 - 10$

$$|x| > -6$$

Step 2: An absolute value **<u>always</u>** gives a **<u>positive</u>** answer, so it will always be greater than any negative number. ALL values of *x* will make this inequality true, so the solution set is all real numbers.

Graph of Solution Set: (<u>entire line is highlighted)</u>

Interval Notation: $(-\infty, \infty)$

Set Builder Notation: $\Re$