Solving Quadratic Equations by Completing the Square

A quadratic equation is an equation in the form of $\mathbf{ax^2 + bx + c = 0}$ where **a**, **b**, and **c** are constants and $\mathbf{a \neq 0}$. One of the methods that can be used to solve a quadratic equation is completing the square.

Step 1:	Isolate the constant if it is not isolated.
Step 2:	If the coefficient of x^2 is not 1, divide both side of the equation by the coefficient of x^2.
Step 3:	Divide the coefficient of x by 2, and square the result.
Step 4:	Add the result from step 3 to both sides of the equation, and simplify.
Step 5:	Factor the variable side of the equation.
	(Hint: In order to have a complete square, both factors must be the same.)
Step 6:	Take the square root of both sides of the equation.
	Note: Place the $\pm$ sign in front of square root of **constant**.
Step 7:	Isolate x.

Example 1: $\mathbf{x^2 - 6x + 5 = 0}$

Step 1: $x^2 - 6x = -5$

Step 2: skip

Step 3: $\frac{-6}{2} = -3$

$(-3)^2 = 9$

Step 4: $x^2 - 6x + 9 = -5 + 9$

$x^2 - 6x + 9 = 4$

Step 5: $(x - 3)(x - 3) = 4$

$(x - 3)^2 = 4$

Hint: -3 is half of the coefficient of x.

Step 6: $\sqrt{(x-3)^2} = \pm\sqrt{4}$

$x - 3 = \pm 2$

Step 7: $x - 3 = -2$ or $x - 3 = 2$

$x = 1$ or $x = 5$

Example 2: $\mathbf{3x^2 + 6x - 72 = 0}$

Step 1: $3x^2 + 6x = 72$

Step 2: $\frac{3x^2}{3} + \frac{6x}{3} = \frac{72}{3}$

$x^2 + 2x = 24$

Step 3: $\frac{2}{2} = 1$

$(1)^2 = 1$

Step 4: $x^2 + 2x + 1 = 24 + 1$

$x^2 + 2x + 1 = 25$

Step 5: $(x + 1)(x + 1) = 25$

$(x + 1)^2 = 25$

Hint: 1 is half of 2 (the coefficient of x).

Step 6: $\sqrt{(x+1)^2} = \pm\sqrt{25}$

$x + 1 = \pm 5$

Step 7: $x + 1 = -5$ or $x + 1 = 5$

$x = -6$ or $x = 4$

Example 3: $x^2 + 6x + 10 = 0$

Step 1: $x^2 + 6x = -10$

Step 2: skip

Step 3: $\frac{6}{2} = 3$

$\left(\frac{6}{2}\right)^2 = 9$

Step 4: $x^2 + 6x + 9 = -10 + 9$

$x^2 + 6x + 9 = -1$

Step 5: $(x + 3)(x + 3) = -1$

$(x + 3)^2 = -1$

Hint: 3 is half of the coefficient of x.

Step 6: $\sqrt{(x+3)^2} = \pm\sqrt{-1}$

$x + 3 = \pm i$

(Note: $\sqrt{-1} = i$)

Step 7: $x + 3 = -i$ or $x + 3 = i$

$x = -3 - i$ or $x = -3 + i$

Example 4: $2x^2 + 5x - \frac{1}{2} = 0$

Step 1: $2x^2 + 5x = \frac{1}{2}$

Step 2: $\frac{2}{2}x^2 + \frac{5}{2}x = \frac{1/2}{2}$

$x^2 + \frac{5}{2}x = \frac{1}{4}$

Step 3: $\frac{5/2}{2} = \left(\frac{5}{2}\right)\left(\frac{1}{2}\right) = \frac{5}{4}$

$\left(\frac{5}{4}\right)^2 = \frac{25}{16}$

Step 4: $x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{1}{4} + \frac{25}{16}$

$x^2 + \frac{5}{2}x + \frac{25}{16} = \frac{29}{16}$

Step 5: $\left(x + \frac{5}{4}\right)\left(x + \frac{5}{4}\right) = \frac{29}{16}$

$\left(x + \frac{5}{4}\right)^2 = \frac{29}{16}$

Hint: $\frac{5}{4}$ is half of $\frac{5}{2}$ (the coefficient of x.)

Step 6: $\sqrt{(x + \frac{5}{4})^2} = \pm\sqrt{\frac{29}{16}}$

$x + \frac{5}{4} = \pm\frac{\sqrt{29}}{4}$

Step 7: $x + \frac{5}{4} = -\frac{\sqrt{29}}{4}$ or $x + \frac{5}{4} = \frac{\sqrt{29}}{4}$

$x = \frac{-5-\sqrt{29}}{4}$ or $x = \frac{-5+\sqrt{29}}{4}$