Divergence of a Vector

Consider a vector **A** (we use **boldface** to denote a vector) with rectangular coordinates A_x, A_y, and A_z. We can write **A** as follows:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

where **i** is a unit vector in the x direction, **j** is a unit vector in the y direction, and **k** is a unit vector in the z direction.

Using $\nabla = \mathbf{i}\,\partial/\partial x + \mathbf{j}\,\partial/\partial y + \mathbf{k}\,\partial/\partial z$, we define the divergence of **A**, written $\nabla \bullet \mathbf{A}$ ("del dot **A**") or div **A**, as follows:

$$\nabla \bullet \mathbf{A} = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z \qquad (1)$$

(For a quick review of partial derivatives, see http://www.math.wisc.edu/~CONRAD/s08/partials.pdf)

Notice that the divergence is a *scalar*. It is the sum of one or more scalars. (It can be thought of as a dot product between the del operator – the ∇ – and the vector **A**, but understanding that point is not essential to success here.)

The following websites may also be helpful:

http://www.tech.plym.ac.uk/maths/resources/pdflatex/div_curl.pdf
http://hyperphysics.phy-astr.gsu.edu/hbase/vvec.html

EXAMPLE:

Suppose $\mathbf{A} = 3x^2\,\mathbf{i} + 5y\,\mathbf{j}$. Compute div **A**.

STEP 1: Compute the necessary partial derivatives

$$\partial A_x/\partial x = 6x \qquad \partial A_y/\partial y = 5 \qquad \partial A_z/\partial z = 0$$

STEP 2: Use equation (1) to compute the divergence

$$\text{Div}\,\mathbf{A} = \nabla \bullet \mathbf{A} = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z = 6x + 5 \qquad (2)$$

***Further* EXAMPLE:**

If you want to know the value of the divergence at any point (x, y, z), you just substitute the values of x, y, and z into the divergence formula.

Find the divergence of the function (2) above at the point x = 2, y = 3.

$$\text{Div } \mathbf{A} = 6x + 5 = (6)(2) + 5 = 17$$

Interpretation

The divergence of a vector field at any point represents how much the field spreads out. For a nice visual, see http://keep2.sjfc.edu/faculty/kgreen/vector/block2/del_op/node5.html.