Solving Log (or Natural Log) Equations

You can have a maximum of one log (ln) on each side of the equal sign. (i.e., You can have zero log (or ln) or one log (or ln) on each side but not more.)

Case I: If there is log (or ln) on only one side of the equal sign

1) Change to exponential form
2) Solve for x

$\log_7 x = 2 \quad \Rightarrow \quad x = 7^2 \quad \Rightarrow \quad x = 49$

Case II: If there is a log (or ln) on both sides of the equal sign

1) You must combine the log (or ln) into a single log (or ln) so that you only have one log (or ln) on each side of the equal sign
2) Drop both log (or ln)…..it will appear as if the log (or ln) cancels out
3) Solve for x

$\ln 3 + \ln(2x+1) = \ln 15$

$\ln[3(2x+1)] = \ln 15$ Step 1

$3(2x+1) = 15$ Step 2

$6x + 3 = 15$

$6x = 12$

$x = 2$

Note: You must *check* answers in log (or ln) problems to make sure the answer lies within the domain. The check is not shown here.

Solving Problems When the Variable is in the Exponent

You log (or ln) both sides of an equation if and only if the variable is in the exponent (unless the equation can be *factored*).

1) If the variable is in the exponent, take the log (or ln) both sides of the equation
2) "Pop" the exponent down in front of the log (or ln)
3) Solve for x (remember that $\ln e = 1$)

$e^{3x} = 4$

$\ln e^{3x} = \ln 4$ Step 1

$3x \ln e = \ln 4$ Step 2

$3x(1) = \ln 4$

$3x = \ln 4$

$x = \dfrac{\ln 4}{3}$

Note: If the equation can be factored, first factor the equation. Then use the method above.

Example:

$e^{2x} - 3e^x + 2 = 0$

$(e^x - 1)(e^x - 2) = 0$

$e^x - 1 = 0$ or $e^x - 2 = 0$

Now use the method shown to the left to solve each equation to give $x = 0$ or $x = \ln 2$.