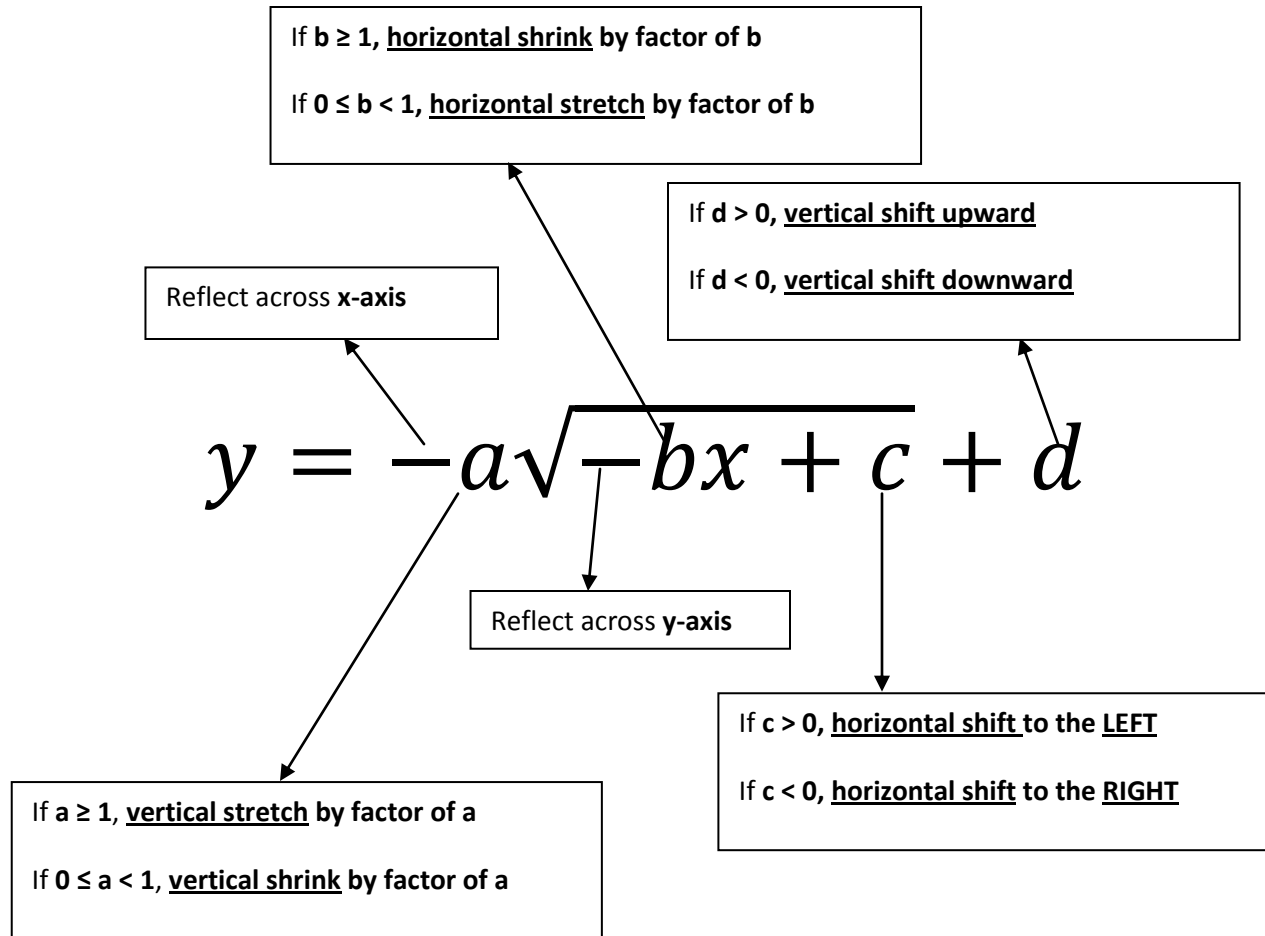


Transformations of Functions



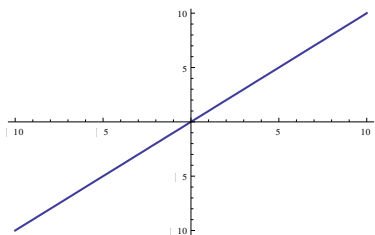
Multiple Transformations

When graphing a function that contains more than one transformation, perform the transformations in the following order:

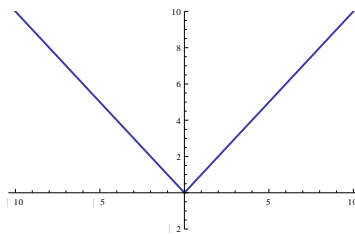
1. Horizontal shifting
2. Stretching or Shrinking
3. Reflection
4. Vertical shifting

Graphs of Basic Functions

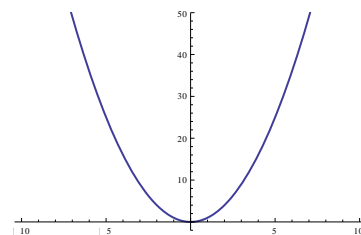
Linear Function
 $y = x$



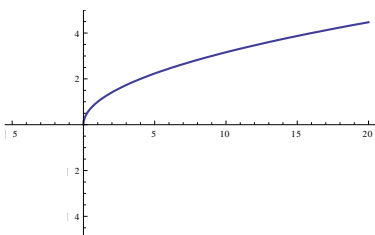
Absolute Function
 $y = |x|$



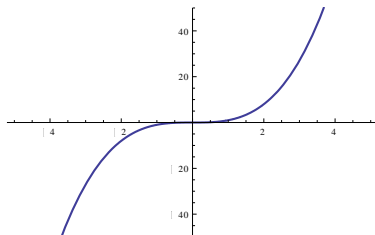
Quadratic Function
 $y = x^2$



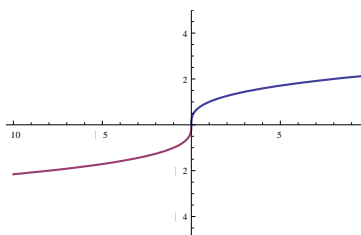
Square Root Function
 $y = \sqrt{x}$



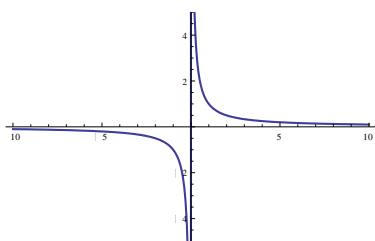
Cube Function
 $y = x^3$



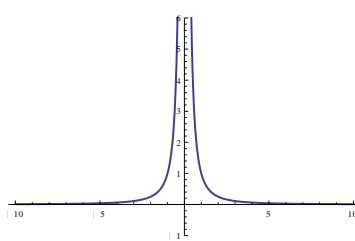
Cube Root Function
 $y = \sqrt[3]{x}$



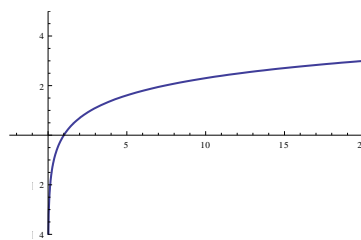
Rational Function
(odd exponent)
 $y = \frac{1}{x}$



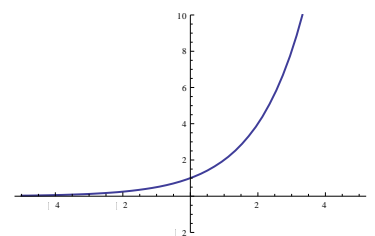
Rational function
(even exponent)
 $y = \frac{1}{x^2}$



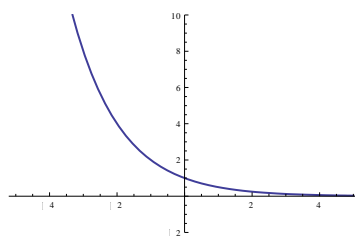
Log Function
 $y = \log x$ and $x > 0$



Exponential Function
 $y = a^x$ if $a > 1$



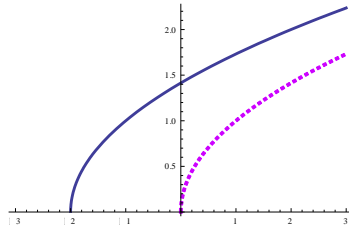
Exponential Function
 $y = a^x$ if $0 < a < 1$



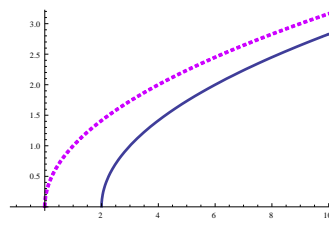
Examples of Transformations

NOTE: Dotted line is original function

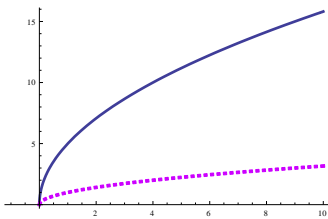
Example: Transformation of $y = \sqrt{x}$



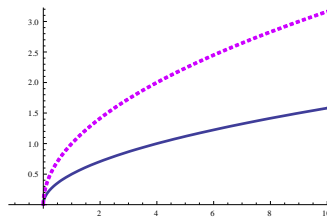
$y = \sqrt{x+2}$
Horizontal shift to the left 2 units



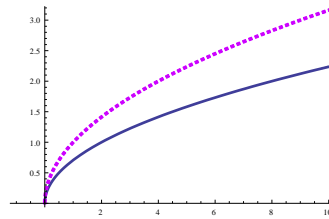
$y = \sqrt{x-2}$
Horizontal shift to the right 2 units



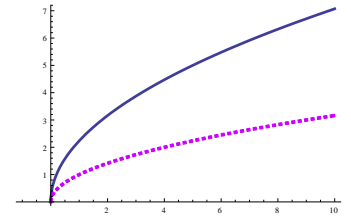
$y = 5\sqrt{x}$
Vertical stretch by factor of 5



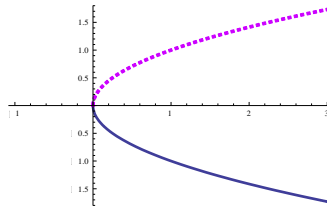
$y = 0.5\sqrt{x}$
Vertical shrink by factor of 0.5



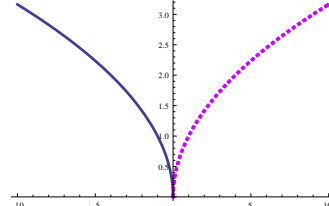
$y = \sqrt{0.5x}$
Horizontal stretch by factor of 0.5



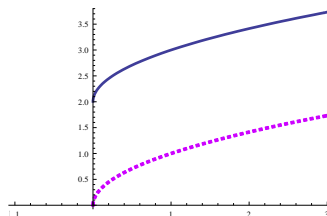
$y = \sqrt{5x}$
Horizontal shrink by factor of 5



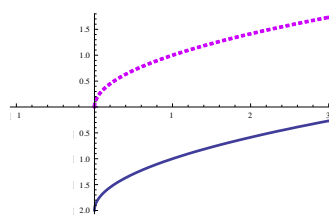
$y = -\sqrt{x}$
Reflect across x-axis



$y = \sqrt{-x}$
Reflect across y-axis



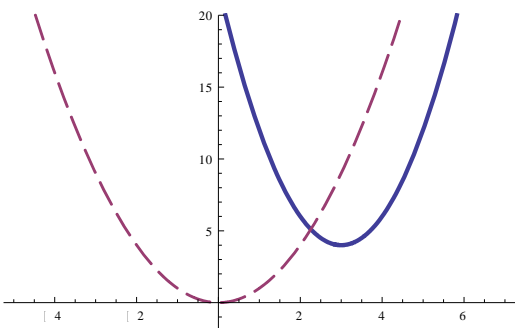
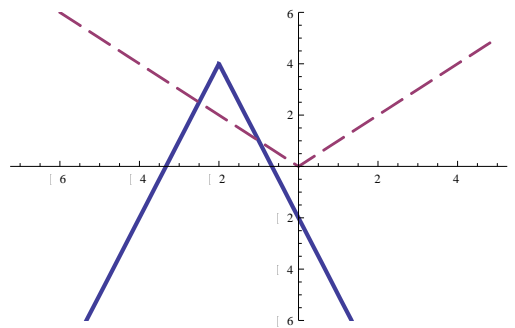
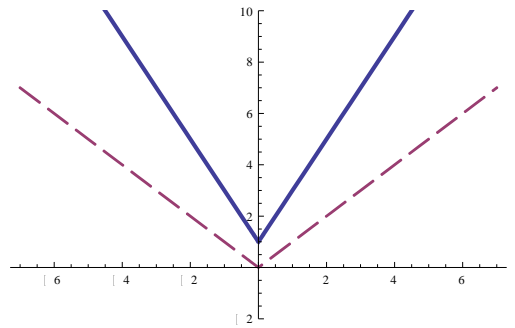
$y = \sqrt{x} + 2$
Vertical shift up 2 units



$y = \sqrt{x} - 2$
Vertical shift down 2 units

Examples of Multiple Transformations

NOTE: Dashed line is original function.

$y_1 = x^2$ $y_2 = 2(x - 3)^2 + 4$ <ol style="list-style-type: none"> 1. Horizontal shift to the right 3 units 2. Vertical stretch by factor of 2 3. No Reflection 4. Vertical shift upward 4 units 	
$y_1 = x $ $y_2 = -3 x + 2 + 4$ <ol style="list-style-type: none"> 1. Horizontal shift to the left 2 units 2. Vertical stretch by factor of 3 3. Reflect across x-axis 4. Vertical shift upward 4 units 	
$y_1 = x $ $y_2 = 2x + 1$ <ol style="list-style-type: none"> 1. No Horizontal shift 2. Horizontal shrink by factor of 2 3. No Reflection 4. Vertical shift upward 1 unit 	
$y_1 = x^2$ $y_2 = -(x + 2)^2 + 3$ <ol style="list-style-type: none"> 1. Horizontal shift to the left 2 units 2. No stretch or shrink 3. Reflect across x-axis 4. Vertical shift upward 3 units 	