

Common Integrals

Basic Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1) \quad \int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + c$$

Exponential and Logarithmic Integrals

$$\int e^x dx = e^x + c \quad \int a^x dx = \frac{a^x}{\ln a} + c \quad \int \ln x dx = x \ln x - x + c$$

Trigonometric Integrals

$$\begin{aligned} \int \sin x dx &= -\cos x + c & \int \cos x dx &= \sin x + c & \int \tan x dx &= \ln|\sec x| + c \\ \int \csc x dx &= -\ln|\csc x + \cot x| + c & \int \sec x dx &= \ln|\sec x + \tan x| + c & \int \cot x dx &= \ln|\sin x| + c \\ \int \csc^2 x dx &= -\cot x + c & \int \sec^2 x dx &= \tan x + c \\ \int \csc x \cot x dx &= -\csc x + c & \int \sec x \tan x dx &= \sec x + c \end{aligned}$$

Hyperbolic Integrals

$$\begin{aligned} \int \sinh x dx &= \cosh x + c & \int \cosh x dx &= \sinh x + c & \int \tanh x dx &= \ln(\cosh x) + c \\ \int \operatorname{csch} x dx &= \ln\left|\tanh \frac{x}{2}\right| + c & \int \operatorname{sech} x dx &= \tan^{-1}|\sinh x| + c & \int \operatorname{coth} x dx &= \ln|\sinh x| + c \\ \int \operatorname{csch}^2 x dx &= -\operatorname{coth} x + c & \int \operatorname{sech}^2 x dx &= \tanh x + c \\ \int \operatorname{csch} x \operatorname{coth} x dx &= -\operatorname{csch} x + c & \int \operatorname{sech} x \tanh x dx &= -\operatorname{sech} x + c \end{aligned}$$

Inverse Trigonometric Integrals

$$\begin{aligned} \int \sin^{-1} x dx &= x \sin^{-1} x + \sqrt{1-x^2} + c & \int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1} x + c \\ \int \cos^{-1} x dx &= x \cos^{-1} x - \sqrt{1-x^2} + c & \int \frac{-1}{\sqrt{1-x^2}} dx &= \cos^{-1} x + c \\ \int \tan^{-1} x dx &= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + c & \int \frac{1}{x^2+1} dx &= \tan^{-1} x + c \end{aligned}$$

Substitution, Integration by Parts, and a Useful Property

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du \quad \text{where } u = g(x)$$

$\int u dv = uv - \int v du$ Note: Select u and dv in such a way that you can differentiate u and integrate dv to find the other quantities.

If $f(x)$ is an **odd** function, then $\int_{-a}^a f(x)dx = 0$.

Note: Check by differentiating.