Rational Inequalities

1) Replace the inequality symbol with an equal sign and solve the resulting equation, obtaining boundary number(s).
2) Set the denominator equal to zero and solve, obtaining another boundary number. (Remember that this number itself cannot be in the solution set since it makes the denominator zero)
3) Place the boundary numbers on the number line and test a point in each region to determine which regions satisfy the inequality.
4) Write the solution set.

Example: Solve $\frac{x}{x+2} \geq 2$

Step #1

$$\frac{x}{x+2} = 2$$

$$\cancel{(x+2)}\frac{x}{\cancel{(x+2)}} = 2(x+2)$$

$$x = 2x + 4$$

$$x = -4$$

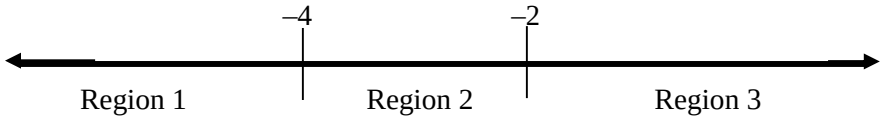
Step #2

$$x + 2 = 0$$

$$x = -2$$

The Boundary Numbers are -4 *and* -2

3. We pick any number in a region and test to see if that region makes the inequality true.
We will test –6 in Region 1, –3 in Region 2, and 0 in Region 3.



Region 1	Region 2	Region 3
$\frac{-6}{-6+2} \geq 2?$	$\frac{-3}{-3+2} \geq 2?$	$\frac{0}{0+2} \geq 2?$
$\frac{-6}{-4} \geq 2?$	$\frac{-3}{-1} \geq 2?$	$0 \geq 2?$
$\frac{3}{2} \geq 2?$	$3 \geq 2?$	$0 \geq 2?$
FALSE	TRUE	FALSE

5. Solution Set: Numbers in Region 2 (but NOT the endpoint -2 (since we cannot divide by zero))

Interval Notation: $[-4, -2)$ **Set Builder Notation**: $\{x| -4 \leq x < -2\}$

Rational Inequalities

Alternate Method

1) Manipulate the inequality so that we have zero on one side.
2) Force the other side of the equation into a single fraction.
3) Boundary numbers are found by setting both numerator and denominator to zero.
4) Determine the **sign** of the fraction in each region of our number line.
5) Graph the "true" intervals on the number line and write the solution set.

Step 1: Get zero on one side

$$\frac{x}{x+2} \geq 2$$

$$\frac{x}{x+2} - 2 \geq 0$$

Step 2: Combine left side into single fraction

$$\frac{x}{x+2} - \frac{2(x+2)}{(x+2)} \geq 0$$

$$\frac{x-2x-4}{(x+2)} \geq 0$$

$$\frac{-x-4}{x+2} \geq 0$$

Step 3: Find the boundary numbers

Numerator: $-x - 4 = 0$

$-x = 4$

$x = -4$

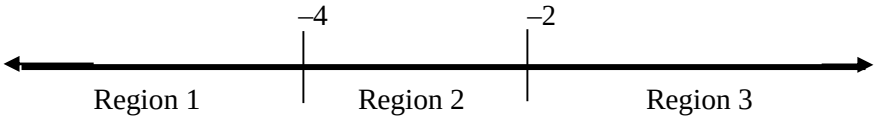
Denominator: $x + 2 = 0$

$x = -2$

Draw number line with these 2 numbers marking the boundaries of our testing zones.

4. We pick any number in a region and test to see what the sign of the quotient is in that region.

We will test –100 in Region 1, –3 in Region 2, and 80 in Region 3.

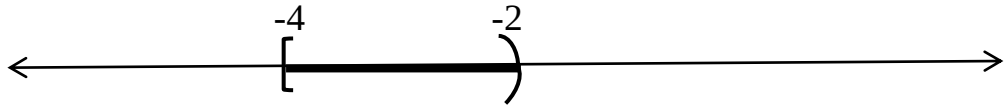


$\frac{-(-100)-4}{-6+2} = \frac{positive}{negative} = negative$ FALSE

$\frac{-(-3)-4}{-6+2} = \frac{negative}{negative} = positive$ TRUE

$\frac{-(80)-4}{-6+2} = \frac{negative}{negative} = positive$ FALSE

5. Solution Set: Numbers in Region 2 (but NOT the endpoint -2 (since we cannot divide by zero))



Interval Notation: $[-4, -2)$ **Set Builder Notation**: $\{x | -4 \leq x < -2\}$