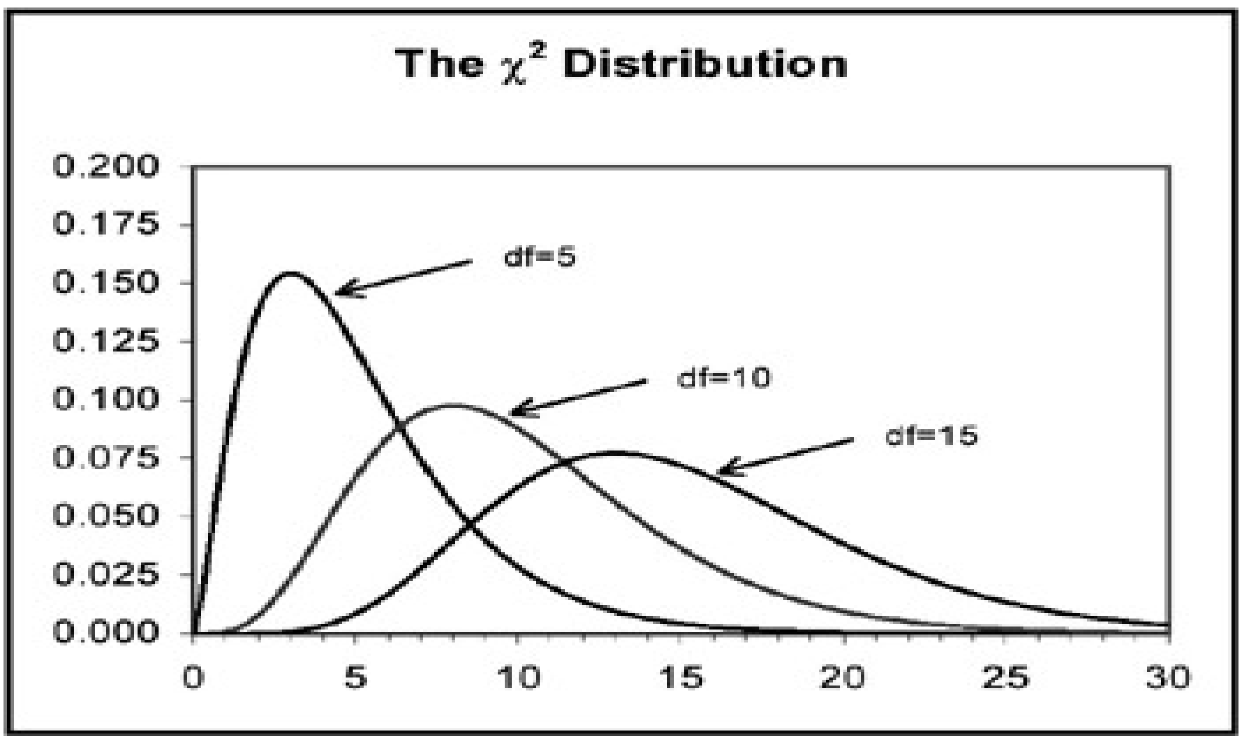
Chi-Square (χ^2) Goodness of Fit Test

The Chi-Square Goodness of Fit is used to determine whether a frequency distribution (observations) follows a specific distribution (expected values).

H_0: Follows the specific distribution
H_1: Does **not** follow the specific distribution

The Chi-Square test is a right tail test. The Chi-Square distribution is right skewed. It is not symmetric. The shape of the Chi-Square distribution is dependent on the degrees of freedom (*k*-1, where *k* is the number of categories).



1. Set up the hypothesis test
   H_0: Follows Distribution (assumed truth)
   H_1: Does not follow Distribution (suspicion)

2. Choose α
   Once the level of significance has been chosen, the Chi-Square critical value must be looked up in the table for the Area to the Right of Critical Value (TABLE VII for Sullivan Fundamentals of statistics).

3. Calculate the test statistic:
   Expected counts are calculated as: $E_i = np_i$
   Where n is the total number of observations and p_i is the expected probability.

$$\chi^2_{test} = \sum \frac{(O_i - E_i)^2}{E_i}$$

for $i = 1,2, \dots, k$

and degrees of freedom $df = k - 1$

   - *all expected frequencies* ≥ 1
   - no more than 20% of frequencies < 5

4. Use the Chi-Square Test Statistic χ^2_{test} and
   the degrees of freedom to estimate the P-value using TABLE VII.

5. Make your decision:
   Reject H_0 if

P-value $< \alpha$	OR	$\chi^2_{test} > \chi^2_{critical}$ (ie., the test value lies within the critical area of the graph)

6. Interpret your decision
   There is sufficient (H_0 $rejected$)/ insufficient (fail to reject H_0) evidence that the frequency distribution does not follow the specified distribution.

Example:

Is a six sided die a fair die?

We roll the die 60 times and observe the following frequency distribution.

Roll	Observed (O)	Expected (E)	Difference (O – E)	$(O - E)^2$	$\frac{(O - E)^2}{E}$
1	3	10	-7	49	$\frac{49}{10}$
2	7	10	-3	9	$\frac{9}{10}$
3	9	10	-1	1	$\frac{1}{10}$
4	11	10	1	1	$\frac{1}{10}$
5	13	10	3	9	$\frac{9}{10}$
6	17	10	7	49	$\frac{49}{10}$

1. H_0: The outcomes from rolling the die follow the uniform distribution
   (or $p = \frac{1}{6}$ or "the die is fair")

   H_1: The die does not follow the uniform distribution
   (or $p \neq \frac{1}{6}$ or "the die is unfair")

2. Decide on $\alpha = 0.05$ level of significance.

3. Calculate the test statistic.

$$\chi^2_{test} = \frac{49}{10} + \frac{9}{10} + \frac{1}{10} + \frac{1}{10} + \frac{9}{10} + \frac{49}{10}$$

$$\chi^2_{test} = 11.8$$

4. From TABLE VII, the $\chi^2_{critical}$ value for an area of 0.05 with 5 degrees of freedom is 11.070.

5. Since $\chi^2_{test} = 11.8 > 11.070 = \chi^2_{critical}$

   Therefore, $\chi^2_{test} > \chi^2_{critical}$, so we reject H_0

6. There is sufficient evidence at the $\alpha = 0.05$ level of confidence to conclude that the die is not fair.

The TI-84 calculator can do this:
Hit the **STAT** key
Choose **1:Edit**
Enter the Observed values into a list and the Expected values into another list.
Hit the **STAT** key
Arrow right or left to **TESTS**
Choose χ^2 **GOF-TEST**
Enter List Name for the Observed Values
Enter List Name for the Expected Values
Then enter the degrees of freedom (Number of categories subtract 1)
Choose **Calculate**
Make your decision based on the P-value

Use the TI-83 calculator to do it this way:
Hit the **STAT** key
Choose **1:Edit**
Enter the Expected values into **L1**
Enter the observed values into **L2**
Move the cursor up to highlight **L3**

Enter **(2nd L1 – 2nd L2)** [x^2] ÷ **2nd L2**
Hit the **STAT** key and arrow to **CALC**
Choose **1: 1-Var Stats 2nd L3** then the **ENTER** key
The value for ∑x is the Chi-Square test statistic
Compare this value to the Chi-Square critical value from TABLE VII to make your decision.