Trigonometric Identities and Formulas

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$ $\sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}$ $\cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}$ $\tan x = \frac{1}{\cot x}$

$\cot x = \frac{\cos x}{\sin x}$ $\tan x = \frac{\sin x}{\cos x}$

Sum and Difference Identities

$\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$
$1 + \tan^2 x = \sec^2 x$
$1 + \cot^2 x = \csc^2 x$

Even-Odd Identities

$\sin(-x) = -\sin x$
$\cos(-x) = \cos x$
$\tan(-x) = -\tan(x)$

Double Angle Formulas

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$
$= 1 - 2\sin^2 x$
$= 2\cos^2 x - 1$

$\tan 2x = \frac{2\tan x}{1 - tan^2 x}$

Half Angle Formulas

$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\left(\frac{x}{2}\right) = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

Sum – to - Product Formulas

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

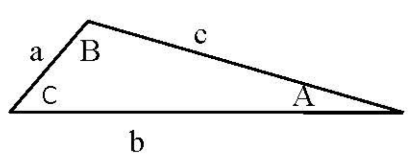
Product – to - Sum Formulas

$2\sin x \cos y = \sin(x+y) + \sin(x-y)$
$2\cos x \sin y = \sin(x = y) - \sin(x-y)$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$
$2\sin x \sin y = \cos(x-y) - \cos(x+y)$

Law of Sines

$\frac{\sin A}{a} = \frac{sin\ B}{b} = \frac{sin\ C}{c}$

Area of a Triangle



$A = \frac{1}{2}ab \sin C$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

The Trigonometric Functions

In a right triangle:

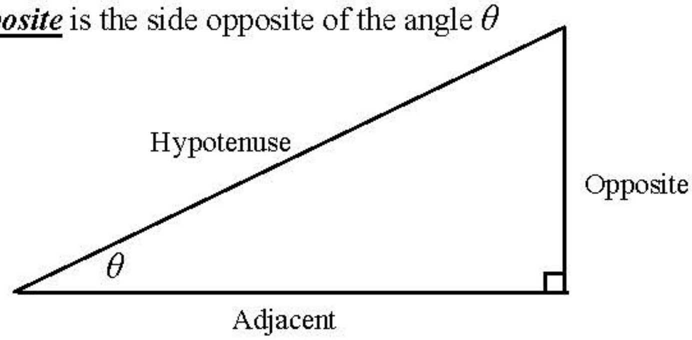
Hypotenuse is the side opposite of the right angle and always is the *longest* side.

Adjacent is the side next to the angle θ. ***Opposite*** is the side opposite of the angle θ

$\sin\theta = \frac{opp}{hyp}$ $\csc\theta = \frac{hyp}{opp}$

$\cos\theta = \frac{adj}{hyp}$ $\sec\theta = \frac{hyp}{adj}$

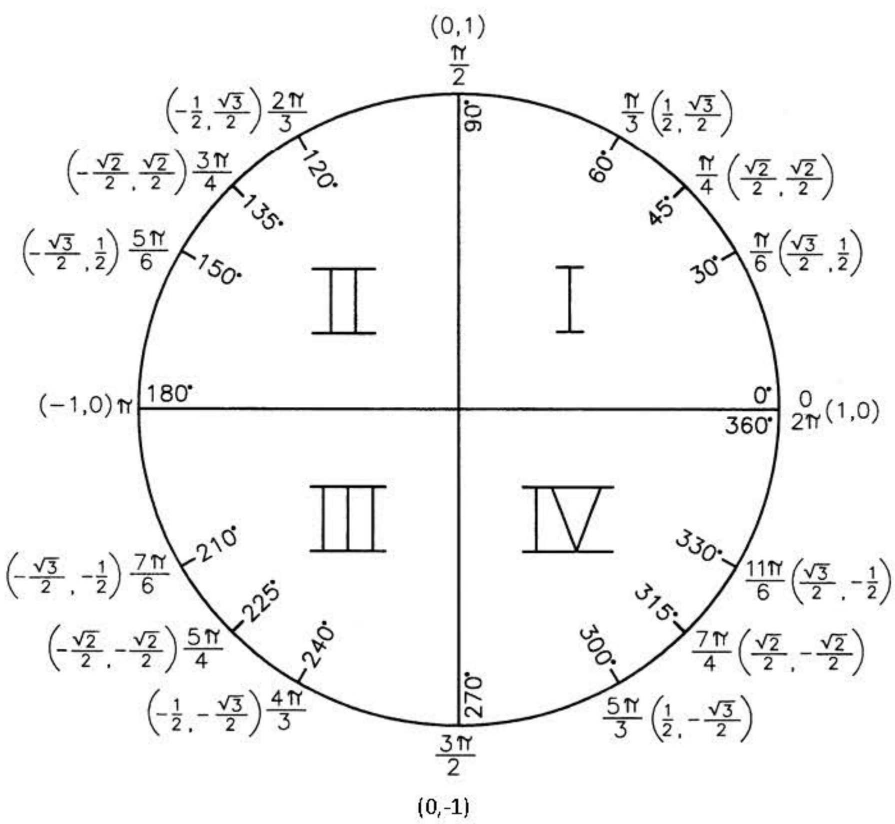
$\tan\theta = \frac{opp}{adj}$ $\cot\theta = \frac{adj}{opp}$



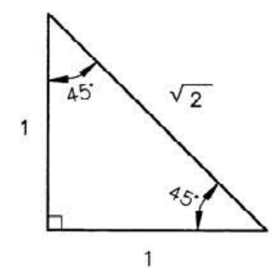
Note: The function in the second column is the reciprocal of the function in the first column.

Unit Circle

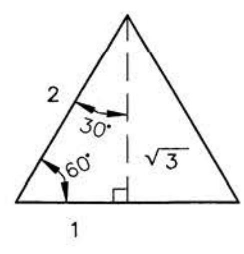
Note: Any point along the unit circle has an x-coordinate whose value is equal to the cosine of the angle and a y-coordinate whose value is equal to the sine of the angle.



Special Triangles



RIGHT TRIANGLE



BISECTED EQUILATERAL TRIANGLE