Vector Addition

Vectors can be described by specifying either their magnitude and direction (polar coordinates) or their x and y components (rectangular, or Cartesian, coordinates). Consider a vector **A** (we use **boldface** to denote a vector) that has a length (or magnitude) A and makes an angle θ with the x-axis. The rectangular coordinates of **A** obey these formulae:

$$A_x = A\cos\theta \tag{1}$$
$$A_y = A\sin\theta \tag{2}$$

$$\text{Magnitude of } \mathbf{A} = |\mathbf{A}| = A = \sqrt{(A_x{}^2 + A_y{}^2)} \tag{3}$$

$$\text{Direction of } \mathbf{A} = \theta = \tan^{-1}(A_y/A_x) \tag{4}$$

Vectors are not added in the same way as ordinary numbers. Vector addition can be done in two ways: *graphically* or *analytically*. Each method involves a specific procedure that must be followed.

For ***Graphical Addition***, please review the applets listed below. It is necessary to use graph paper and a protractor to ensure that vectors are drawn to scale and at appropriate angles.

https://www.msu.edu/~brechtjo/physics/vectorAdd/vectorAdd.html
http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=51
http://www.walter-fendt.de/ph14e/resultant.htm
http://comp.uark.edu/~jgeabana/java/VectorCalc.html

For ***Analytical Addition***, use the following rule:

"The component of the sum equals the sum of the components. "

That is, if **C** = **A** + **B**, then

$$C_x = A_x + B_x \quad \text{and} \quad C_y = A_y + B_y \tag{5}$$

The magnitude and direction of **C** are then defined as follows:

$$\text{Magnitude of } \mathbf{C} = |\mathbf{C}| = C = \sqrt{(C_x{}^2 + C_y{}^2)} \tag{6}$$

$$\text{Direction of } \mathbf{C} = \tan^{-1}(C_y/C_x) \tag{7}$$

<u>EXAMPLE</u>:

Suppose **A** = 3**i** + 5**j** and **B** = 4**i** – 2**j**. If **C** = **A** + **B**, determine the magnitude and direction of **C**.

STEP 1: Identify the components of the individual vectors

$A_x = 3$ $A_y = 5$ $B_x = 4$ $B_y = -2$

STEP 2: Use equation (5) to compute the x and y components of the sum

$C_x = A_x + B_x = 3 + 4 = 7$ $C_y = A_y + B_y = 5 + (-2) = 3$

STEP 3: Use equation (6) to compute the magnitude of the sum

Magnitude of $\mathbf{C} = \sqrt{(7^2 + 3^2)} = \sqrt{49 + 9} = \sqrt{58} = 7.62$

STEP 4: Use equation (7) to compute the direction of the sum

Direction of $\mathbf{C} = \tan^{-1}(3/7) = 23.2°$

Three or more vectors are added just as easily as two. When doing the addition analytically, just add up all the x components to find the total x component, and likewise for y. When doing the addition graphically, the only difference is that you add the vectors two at a time. In other words, add the first two and find the resultant; then add the third vector to this resultant to find a new resultant; then repeat the process until finished.

The *equilibrant* of a vector is the one that cancels it out – that is, the vector that when added to your original vector gives a resultant (sum) of zero. The equilibrant of any vector has the same magnitude but the opposite direction. Sometimes in vector algebra, you may see the equilibrant referred to as the *negative* of a vector. If you have a vector **A**, then the equilibrant may be written as –**A**.

So if **A** has components A_x and A_y, then the equilibrant –**A** has components $-A_x$ and $-A_y$.