Vector Dot Product

Consider a vector **A** (we use **boldface** to denote a vector) with rectangular coordinates A_x and A_y. We can write A as follows:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$$

where **i** is a unit vector in the x direction and **j** is a unit vector in the y direction.

If we have two vectors **A** and **B**, defined as above, then we define the *dot product* of **A** and **B**, written **A • B** (read "A dot B") as follows:

$$\mathbf{A} \bullet \mathbf{B} = A_xB_x + A_yB_y \qquad (1)$$

Notice that the dot product is a *scalar*; it is the sum of one or more scalars. It may be referred to as the *scalar product*.

The following websites may also be helpful:

http://www.falstad.com/dotproduct/
http://betterexplained.com/articles/vector-calculus-understanding-the-dot-product/

***Simple* EXAMPLE**:

Suppose **A** = 3**i** + 5**j** and **B** = 4**i** – 2**j**

STEP 1: Identify the components of each vector

$A_x = 3$ $\quad A_y = 5$ $\quad B_x = 4$ $\quad B_y = -2$

STEP 2: Use equation (1) to compute the dot product

$$\mathbf{A} \bullet \mathbf{B} = A_xB_x + A_yB_y = (3)(4) + (5)(-2) = 12 - 10 = 2$$

***More Complicated* EXAMPLE**:

The rules are the same for three-dimensional vectors.

Suppose **A** = 3**i** + 5**j** + 8**k** and **B** = 4**i** – 2**j** + **k**. Compute **A • B**.

STEP 1: Identify the components of each vector

$A_x = 3$ $\quad A_y = 5$ $\quad A_z = 8$ $\quad B_x = 4$ $\quad B_y = -2\, B_z = 1$

STEP 2: Use equation (1) to compute the dot product

$$\mathbf{A} \bullet \mathbf{B} = A_x B_x + A_y B_y + A_z B_z = (3)(4) + (5)(-2) + (8)(1) = 10$$

Alternate Formula

It happens that the dot product can also be calculated as follows:

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}|\ |\mathbf{B}| \cos\theta, \qquad (2)$$

where $|\mathbf{A}|$ is the magnitude of **A**, $|\mathbf{B}|$ is the magnitude of **B**, and θ is the angle between them.

This formula may be helpful when you are given the magnitudes and angles to begin with.

EXAMPLE:

Suppose A is a vector of magnitude 6 at an angle of 60°. Suppose B is a vector of magnitude 4 at an angle of -25°. Compute **A • B.**

STEP 1: Compute the angle θ between the vectors

$$\theta = 60° - (-25°) = 85°$$

STEP 2: Compute the cosine of the angle θ

$$\cos\theta = \cos(85°) \sim 0.0872 \text{ (rounding off)}$$

STEP 3: Use equation (2) to compute the dot product

$$\mathbf{A} \bullet \mathbf{B} = |\mathbf{A}|\ |\mathbf{B}| \cos\theta = (6)(4)(0.0872) \sim 2.09$$

Interpretation

The dot product of A and B can be thought of as the product of one vector times the component of the second that lies parallel to the first.

Note that **A • B** ranges from 0 when **A** and **B** are perpendicular to $|\mathbf{A}|\ |\mathbf{B}|$ when **A** and **B** are parallel.