

Work Problems

Here are a few tips to remember when solving work problems:

1. The units of measure must be the same. For example, you can't use both hours and minutes for time in the same problem. You must convert **all** times into hours **or** minutes.
2. For each worker involved, find the fractional part of the job that can be done in one unit of time, such as 1 minute or 1 hour. For example, if a person can do a complete job in 3 days, he can do one-third of it in 1 day. We'll call this fraction **the rate of work**.
3. The rate of work that one person can do plus the rate of work that another person can do is equal to the rate of work that the two can do together. Example: If Bill can build one-third of the dog house in 1 day and Gary can build one-fifth of it in 1 day, together they can build one-third plus one-fifth of the dog house in 1 day.

Example: Mark can dig a ditch in 4 hours. Greg can dig the same ditch in 3 hours. How long would it take them to dig it together?

Solution: Let x = number of hours to dig the ditch together.

If Mark takes 4 hours to dig the ditch, he can dig one-fourth of it in 1 hour. Mark's rate is then $\frac{1}{4}$.

Since Greg can dig the same ditch in 3 hours, then he can dig one-third of it in 1 hour. Greg's rate is then $\frac{1}{3}$. If it takes them x hours to dig it together, they can dig $\frac{1}{x}$ of the ditch in 1 hour.

Equation: Take the sum of the individual rates and set it equal to the combined rate, then solve.

$$\frac{1}{3} + \frac{1}{4} = \frac{1}{x}$$

Multiply by $12x$ (**LCD**) to clear fractions.

$$4x + 3x = 12$$

$$7x = 12$$

$$x = \frac{12}{7}$$

or $x = 1\frac{5}{7}$

It would take them $1\frac{5}{7}$ hrs to dig the ditch.

Sample Problems:

1. If Lisa can type a paper in 5 hours and together she and Bill can type it in 2 hours, how long would it take Bill to type the same paper alone?

Solution: Let x = number of hours that Bill can type the paper.

$$\text{Lisa's rate} = \frac{1}{5} \qquad \text{Bill's rate} = \frac{1}{x} \qquad \text{Combined rate} = \frac{1}{2}$$

Equation: Take the sum of the individual rates and set it equal to the combined rate, then solve.

$$\frac{1}{5} + \frac{1}{x} = \frac{1}{2}$$

Multiply by $10x$ (**LCD**) to clear fractions.

$$\begin{aligned} 2x + 10 &= 5x \\ -3x &= -10 \\ x &= \frac{10}{3} \end{aligned}$$

or $x = 3\frac{1}{3}$

It would take Bill $3\frac{1}{3}$ *hrs* to type the same paper.

2. A swimming pool can be filled by an inlet pipe in 10 hours and emptied by an outlet pipe in 12 hours. One day the pool is empty and the owner opens the inlet pipe to fill the pool, but he forgets to close the outlet pipe. With both pipes open, how long will it take to fill the pool?

Solution: Let x = number of hours it will take for the pool to fill.

$$\text{Inlet pipe rate} = \frac{1}{10} \qquad \text{Outlet pipe rate} = -\frac{1}{12} \qquad \text{Combined rate} = \frac{1}{x}$$

Equation: Take the sum of the individual rates and set it equal to the combined rate, then solve.

$$\frac{1}{10} - \frac{1}{12} = \frac{1}{x}$$

The outlet pipe rate is negative because this is the rate of which water is flowing **out** of the pool.

$$\begin{aligned} 6x - 5x &= 60 \\ x &= 60 \end{aligned}$$

It will take **60 hours** for the pool to fill.